Int J Theor Phys (2010) 49: 31–41
DOI 10.1007/s10773-009-0175-y

Quantum Parrondo's Games Under Decoherence

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Received: 9 May 2009 / Accepted: 15 October 2009 / Published online: 4 November 2009

Abstract We study the effect of quantum noise on history dependent quantum Parrondo's games by taking into account different noise channels. Our calculations show that entanglement can play a crucial role in quantum Parrondo's games. It is seen that for the maximally entangled initial state in the presence of decoherence, the quantum phases strongly influence the payoffs for various sequences of the game. The effect of amplitude damping channel leads to winning payoffs. Whereas the depolarizing and phase damping channels lead to the losing payoffs. In case of amplitude damping channel, the payoffs are enhanced in the presence of decoherence for the sequence AAB. This is because the quantum phases interfere constructively which leads to the quantum enhancement of the payoffs in comparison to the undecohered case. It is also seen that the quantum phase angles damp the payoffs significantly in the presence of decoherence. Furthermore, it is seen that for multiple games of sequence AAB, under the influence of amplitude damping channel, the game still remains a winning game. However, the quantum enhancement reduces in comparison to the single game of sequence AAB because of the destructive interference of phase dependent terms. In case of depolarizing channel, the game becomes a loosing game. It is seen that for the game sequence B the game is loosing one and the behavior of sequences B and BB is similar for amplitude damping and depolarizing channels. In addition, the repeated games of A are only influenced by the amplitude damping channel and the game remains a losing game. Furthermore, it is also seen that for any sequence when played in series, the phase damping channel does not influence the game.

Keywords Quantum Parrondo's games · Decoherence · Payoffs

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1 Introduction

Game theory [1] has been implemented for diverse applications in different areas for example; economics, evolutionary biology, psychology and physics. More recently, the game theory is being used to model distributed and parallel computing in the field of computer science. It is the theory of decision making and conflict between different agents. Starting from the works of Meyer and Eisert, quantum game theory has been recognized as an important theory with useful applications [2–11].

In quantum game theory, the initial state entanglement has produced interesting results. Quantum entanglement is one of the fascinating features of quantum mechanics and plays a crucial role in quantum information processing as well. When quantum information processing is performed in the real world, the decoherence caused by an external environment is inevitable. Decoherence effects in different quantum games have been studied in Refs. [5, 12, 13]. Here in this work, we are interested to study the decoherence effects on the history dependent quantum Parrondo's games played in various sequences.

In the context of classical Parrondo's games, the two games that are losing when played individually can be combined in various sequences to produce a winning game [14, 15]. Parrondo's games have attracted considerable attention in the past as they can be related to physical systems such as the Brownian ratchet [16], lattice gas automata [17] and spin systems [18]. Based on the maximal entanglement between the qubits, a quantization protocol for the history dependent Parrondo's games was proposed by Flitney et al. [19]. Multi-player extension to classical Parrondo's games was given by Toral [20].

In this paper, we study the effect of decoherence on history dependent quantum Parrondo's games by considering different prototype channels such as amplitude damping, depolarizing and phase damping channels, parameterized by the decoherence parameter $p \in [0, 1]$. The lower and upper limits of the decoherence parameter p correspond to a fully coherent and fully decohered systems, respectively. We study the effect of quantum decoherence on the game dynamics. It is seen that the payoffs are enhanced due to the presence of decoherence in case of amplitude damping channel for the single game of sequence AAB. The enhancement in payoffs occurs due to the constructive interference of quantum phases, δ and β_i. It is also seen that the increase in payoffs for phase damping and depolarizing channels is not much prominent in comparison to the amplitude damping channel. We also analyze the influence of decoherence by playing other sequences such as B, BB, BBB, $AA\ldots A$ and multiple games of sequence AAB. The results are discussed in detail in the results and discussions section.

2 History Dependent Quantum Parrondo's Games

History dependent Parrondo's games consist of two simpler coin tossing games; A and B. Game A is straight forward one player biased coin flipping game that wins 1 when lands head up and loses 1 when it lands tail up. However, game B consists of four biased coins, the selection of each of them depends on history of the games (the results of previous two games). In the classical version, the winning probabilities for coin A and for each coin of game B are given by

$$p_0 = \frac{1}{2} - \epsilon, \qquad p_1 = \frac{7}{10} - \epsilon, \qquad p_2 = p_3 = \frac{1}{4} - \epsilon, \qquad p_4 = \frac{9}{10} - \epsilon \tag{1}$$

respectively. It is shown in Ref. [16] that for a small positive value of ϵ each of the games A and B played individually is a losing game, however, if they are played in various sequences of A and B produce a winning result.

The game A can be quantized by replacing the tossing of a coin by an $SU(2)$ operator on a qubit as given in Ref. [19]

$$A(\theta,\gamma,\delta)=\begin{bmatrix}\exp\left(\frac{-i(\gamma+\delta)}{2}\right)\cos\theta & -\exp\left(\frac{-i(\gamma-\delta)}{2}\right)\sin\theta\\ \exp\left(\frac{i(\gamma-\delta)}{2}\right)\sin\theta & \exp\left(\frac{i(\gamma+\delta)}{2}\right)\cos\theta\end{bmatrix} \tag{2}$$

where $\theta\in[-\pi,\pi]$, γ and $\delta\in[0,2\pi]$. Similarly, the operator for game B consists of four $SU(2)$ operations, each of the form given in (2), where the choice of the use of these operations depends on the outcome of the previous two games:

$$B=\begin{bmatrix}A_1 & 0 & 0 & 0\\ 0 & A_2 & 0 & 0\\ 0 & 0 & A_3 & 0\\ 0 & 0 & 0 & A_4\end{bmatrix} \tag{3}$$

with $A_i=A(\phi_i,\alpha_i,\beta_i)$. The operator B acts on the following three qubits state

$$|\Psi_2\rangle\otimes|\Psi_1\rangle\otimes|\Psi_0\rangle \tag{4}$$

where $|\Psi_2\rangle$ and $|\Psi_1\rangle$ stand for the results of two successive previous games and $|\Psi_0\rangle$ stands for the target qubit that results in some output, say b. Each qubit in (4) could be in one of the possible states of a qubit. The final density matrix of the game is given by

$$\rho_f=U\rho_i U^\dagger \tag{5}$$

For n successive games of sequence B, the operator U can be written as

$$\begin{aligned}U_B&=\left(I^{\otimes n-1}\otimes B\right)\left(I^{\otimes n-2}\otimes B\otimes I\right)\left(I^{\otimes n-3}\otimes B\otimes I^{\otimes 2}\right)\\ &\quad\cdots\left(I\otimes B\otimes I^{\otimes n-2}\right)\left(B\otimes I^{\otimes n-1}\right)\end{aligned} \tag{6}$$

where I represents the single qubit identity operator. Similarly, the operator U for n games of sequence AAB can be written as

$$\begin{aligned}U^n_{AAB}&=\left(I^{\otimes 3n-3}\otimes(B(A\otimes A\otimes I))\right)\left(I^{\otimes 3n-6}\otimes(B(A\otimes A\otimes I))\otimes I^{\otimes 3}\right)\\ &\quad\cdots\left((B(A\otimes A\otimes I))\otimes I^{\otimes 3n-3}\right)\\ &=U^{\otimes n}\end{aligned} \tag{7}$$

where $U=B(A\otimes A\otimes I)$ stands for the operator of a single game sequence AAB. Here in this paper, we consider various sequences of Parrondo's games such as $AA\ldots A$, B, BB, BBB, AAB and a series of AAB. To study the effect of decoherence, we restrict our calculations only to the maximally entangled initial state of the form

$$|\Psi_i\rangle=\frac{1}{\sqrt{2}}(|00\ldots 0\rangle+|11\ldots 1\rangle) \tag{8}$$

We consider $|0\rangle$ as "loss" state and the $|1\rangle$ as a "win" state. Furthermore, we can fix the computational basis of the Hilbert Space for example $\mathcal{H}^{\otimes 3}$ for a single game sequence AAB in the basis ordered as $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$ and $|111\rangle$, respectively.

Table 1 Single qubit Kraus operators for typical noise channels such as depolarizing, amplitude damping and phase damping channels where p represents the decoherence parameter

Depolarizing channel	$E_0=\sqrt{1-3p/4}I,\ E_1=\sqrt{p/4}\sigma_x$ $E_2=\sqrt{p/4}\sigma_y,\quad E_3=\sqrt{p/4}\sigma_z$
Amplitude damping channel	$E_0=\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-p}\end{bmatrix}, E_1=\begin{bmatrix}0 & \sqrt{p}\\ 0 & 0\end{bmatrix}$
Phase damping channel	$E_0=\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-p}\end{bmatrix}, E_1=\begin{bmatrix}0 & 0\\ 0 & \sqrt{p}\end{bmatrix}$

3 Quantum Channels

A natural way to describe the dynamics of a quantum system is to consider it as arising from an interaction between the system and the environment. In general quantum systems are prone to decoherence effects and it is important to analyze these effects in real practical situations. Environmental interactions can destroy the important features of quantum computation. However, quantum error correction [21] and decoherence free subspaces [22] can be used to perform quantum computing even in the presence of noise. In the most general case, the quantum evolution can be described by the superoperator Φ, which can be expressed in Kraus operator representation as [23]

$$\Phi(\rho)=\sum_k E_k\rho E_k^\dagger \tag{9}$$

where

$$\sum_k E_k^\dagger E_k = I \tag{10}$$

Superoperators provide a way to describe the evolution of quantum states in a noisy environment. In our scheme, the Kraus operators are of the dimension 2^3. They are constructed from one qubit operators by taking their tensor product over all n^3 combinations of $\pi(i)$ indices

$$E_k=\left(\bigotimes_\pi\right)e_{\pi(i)} \tag{11}$$

where n is the number of Kraus operators for a single qubit channel. The single qubit Kraus operators for quantum channels considered in this paper are given in Table 1.

4 Results and Discussions

In this section we present our results for different sequences of Parrondo's games in the presence of various noisy channels. The final density matrix of the game after the action of a channel (as specified in (9)) for example, for a single game sequence AAB, can be written as

$$\rho^{AAB}=\Phi\rho_i^{AAB} \tag{12}$$

where $\rho_i^{AAB}=|\Psi^{AAB}\rangle\langle\Psi^{AAB}|$ and $|\Psi^{AAB}\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$. The game's final density matrix after the application of operator U can be computed by using (5). To determine the payoff, we assume that the payoff for a $|1\rangle$ state is $+1$, and is -1 for a $|0\rangle$ state. The total payoff can be determined by using the relation

$$\langle \$ \rangle = \sum_{ijk} (l+m+n)\rho_{ijk} \tag{13}$$

where $l=(-1)^{i+1},\ m=(-1)^{j+1},\ n=(-1)^{k+1}$, the indices i, j and k run from 0 to 1 and ρ_{ijk} represent the diagonal elements of the final density matrix.

The payoff for amplitude damping channel for the game sequence AAB can be written as

$$\begin{aligned}
\langle \$^{\mathrm{AD}} \rangle &= 3p + \cos^2\phi_1 - \cos^2\phi_4 + p[\{-4+(5-2p)p\}\cos^2\phi_1 \\
&\quad + (-1+2p)\{(-1+p)(\cos^2\phi_2 + \cos^2\phi_3) - p\cos^2\phi_4\}] \\
&\quad + \cos^4\theta\{p(-3+6p-4p^2)\cos 2\phi_1 + p(3-6p+4p^2)\cos 2\phi_2 \\
&\quad + 2p(3-6p+4p^2)\cos^2\phi_3 - 2p(3-6p+4p^2)\cos^2\phi_4 \\
&\quad + (1-p)^{3/2}(-\cos(2\delta+\beta_1)\sin 2\phi_1 + \cos(2\delta+\beta_2)\sin 2\phi_2 \\
&\quad + \cos(2\delta+\beta_3)\sin 2\phi_3 - \cos(2\delta+\beta_4)\sin 2\phi_4)\} \\
&\quad + \cos^2\theta[-4p + 2(1-2p)^2(-1+p)\cos^2\phi_1 - 2p(3-6p+4p^2)\cos^2\phi_2 \\
&\quad - 2p(3-6p+4p^2)\cos^2\phi_3 + 2\cos^2\phi_4 + 2(1-2p)^2 p\cos^2\phi_4 \\
&\quad + \sqrt{1-p}\cos(2\delta+\beta_1)\sin 2\phi_1 + \sqrt{1-p}\{-p\cos(2\delta+\beta_1)\sin 2\phi_1 \\
&\quad + (-1+p)(\cos(2\delta+\beta_2)\sin 2\phi_2 + \cos(2\delta+\beta_3)\sin 2\phi_3 \\
&\quad - \cos(2\delta+\beta_4)\sin 2\phi_4)\}]
\end{aligned} \tag{14}$$

The payoff obtained for the case of depolarizing channel is given as

$$\begin{aligned}
\langle \$^{\mathrm{DP}} \rangle &= (-1+p)^2(\cos 2\theta(-\cos^2\phi_1 + \cos^2\phi_4) + (-1+p)\cos^2\theta\sin^2\theta \\
&\quad \times (-\cos(2\delta+\beta_1)\sin 2\phi_1 + \cos(2\delta+\beta_2)\sin 2\phi_2 \\
&\quad + \cos(2\delta+\beta_3)\sin 2\phi_3 - \cos(2\delta+\beta_4)\sin 2\phi_4))
\end{aligned} \tag{15}$$

The payoff in case of phase damping channel becomes

$$\begin{aligned}
\langle \$^{\mathrm{PD}} \rangle &= \cos 2\theta(-\cos^2\phi_1 + \cos^2\phi_4) \\
&\quad - (1-p)^{3/2}\cos^2\theta\sin^2\theta(-\cos(2\delta+\beta_1)\sin 2\phi_1 \\
&\quad + \cos(2\delta+\beta_2)\sin 2\phi_2 + \cos(2\delta+\beta_3)\sin 2\phi_3 \\
&\quad - \cos(2\delta+\beta_4)\sin 2\phi_4)
\end{aligned} \tag{16}$$

where the superscripts AD, DP and PD represent the amplitude damping, depolarizing and phase damping channels, respectively. The winning probability for a flip of a coin is equal to the square of the sine of the rotation angle (i.e. θ and ϕ_i). Whereas δ and β_i's represent the quantum phases. It can be easily checked that the result of Ref. [19] for maximally entangled state can be reproduced by setting the decoherence parameter $p=0$ in (14), (15) and (16). The presence of phase angles δ and β_i's in the (14), (15) and (16) leads to a range of payoffs for the set of values of θ and ϕ_i's corresponding to the classical winning probabilities of the two games as given in (1).

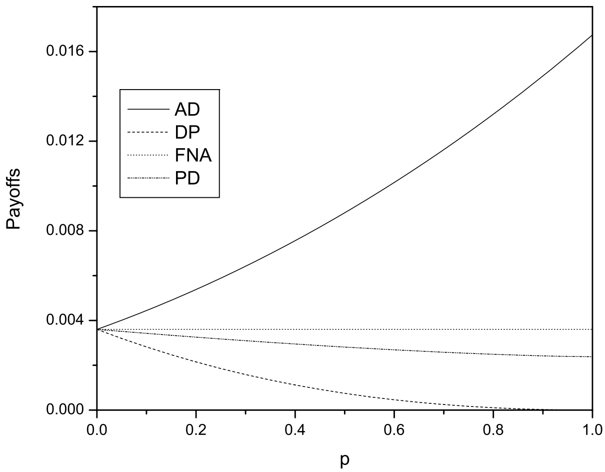


Fig. 1 The expected payoffs for a single game of sequence AAB are plotted as a function of the decoherence parameter, p for amplitude damping channel (AD), depolarizing channel (DP), phase damping channel (PD) and Flitney and Abbott results (FNA) with $\delta = \frac{\pi}{5}$, $\beta_1 = \frac{\pi}{2}$, $\beta_2 = \frac{\pi}{2}$, $\beta_3 = \frac{\pi}{4}$, $\beta_4 = \frac{\pi}{3}$ and $\varepsilon = \frac{1}{168}$

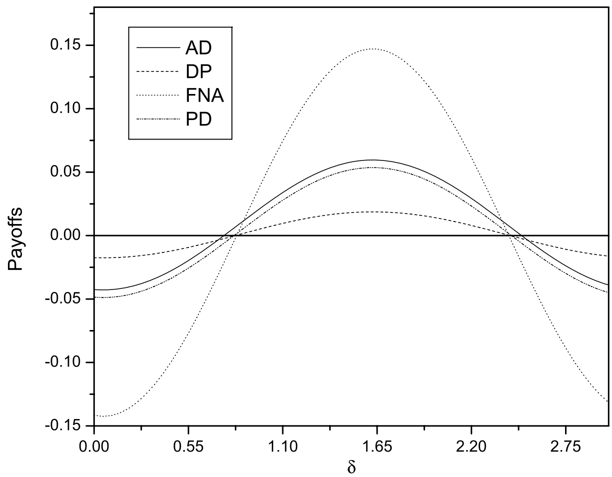


Fig. 2 The expected payoffs for a single game of sequence AAB are plotted as a function of the phase angle, δ for amplitude damping channel, depolarizing channel, phase damping channel and FNA results with $p = 0.5$, $\beta_1 = \frac{\pi}{2}$, $\beta_2 = \frac{\pi}{3}$, $\beta_3 = \frac{\pi}{4}$, $\beta_4 = \frac{\pi}{3}$ and $\varepsilon = \frac{1}{168}$

In Fig. 1, we have plotted the payoffs for the single sequence of game AAB as a function of decoherence parameter, p, for different noise channels. The rotation angles correspond to the classical winning probabilities and the quantum phase angles are taken as $\beta_1 = \beta_2 = \pi/2$, $\beta_3 = \pi/4$, $\beta_4 = \pi/3$, and $\delta = \pi/5$. It is seen that the payoff of the game is further enhanced under decoherence for the amplitude damping channel. The significant increase in payoff for amplitude damping channel results from the constructive interference arising due to the presence of decoherence and the quantum phases. However, under the action of depolarizing and phase damping channels, destructive interference is seen and the game's sequence AAB becomes a losing one. In Fig. 2, the payoffs as a function of quantum phase angle δ are plotted for all the three channels. It is seen that for a particular choice of values of the parameters, the payoffs become positive only for the range $\frac{\pi}{4} \lesssim \delta \lesssim \frac{3\pi}{4}$ and varies symmetrically around $\frac{\pi}{2}$. The payoffs are also significantly damped in comparison to the undecohered case. In Fig. 3 we have plotted the payoffs as a function of quantum phase angle β_1. It is seen that the game becomes a winning game. The effect of quantum phase angle β_2 on the payoffs is shown in Fig. 4. In Figs. 5 and 6, we have plotted the payoffs

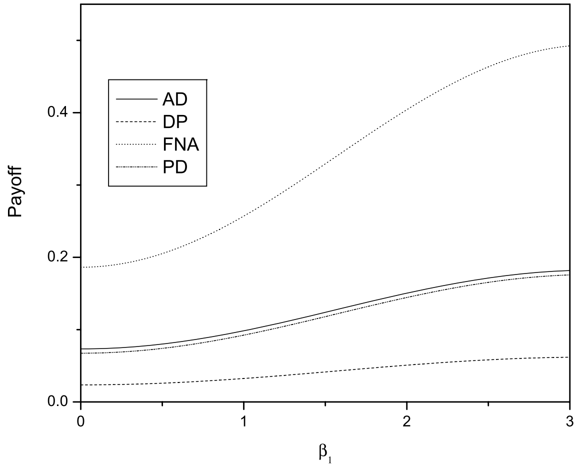


Fig. 3 The expected payoffs for a single game of sequence AAB are plotted as a function of the quantum phase angle, β_1 for amplitude damping channel, depolarizing channel, phase damping channel and FNA results with $p = 0.5$, $\delta = \frac{\pi}{2}$, $\beta_2 = \frac{\pi}{3}$, $\beta_3 = \frac{\pi}{2}$, $\beta_4 = \pi$ and $\varepsilon = \frac{1}{168}$ respectively

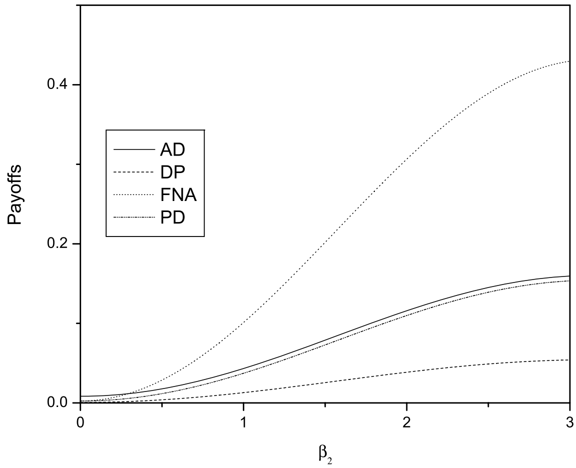


Fig. 4 The expected payoffs for a single game of sequence AAB are plotted as a function of the quantum phase angle, β_2 for amplitude damping channel, depolarizing channel, phase damping channel and FNA results with $p = 0.5$, $\delta = \pi$, $\beta_1 = \frac{\pi}{2}$, $\beta_3 = \pi$, $\beta_4 = \frac{\pi}{2}$ and $\varepsilon = \frac{1}{168}$

versus the phase angles β_3 and β_4 respectively. It is shown that the game behaves as a loosing and winning one respectively. Similar to the behavior of the game against the phase angle δ, the payoffs are significantly damped against the phase angles β_i's under decoherence.

In Fig. 7, we have plotted the expected payoffs as a function of decoherence under the influence of various noisy channels by playing the sequence AAB repeatedly at $\varepsilon = \frac{1}{168}$, where the inset figure corresponds to $\varepsilon = \frac{1}{112}$. The expected payoffs for this generalized case for different channels are obtained as

$$\begin{aligned}
\langle \$^{\mathrm{AD}} \rangle &= \left(\frac{1}{60} p + \left(\frac{2}{15} - 2.27p + 0.27p^2 \right) \varepsilon \right) \\
\langle \$^{\mathrm{DP}} \rangle &= \left(\frac{2}{15} - 0.35p + 0.24p^2 \right) \varepsilon \\
\langle \$^{\mathrm{PD}} \rangle &= \frac{2}{15} \varepsilon
\end{aligned} \tag{17}$$

Fig. 5 The expected payoffs for a single game of sequence AAB are plotted as a function of the quantum phase angle, β_3 for amplitude damping channel, depolarizing channel, phase damping channel and FNA results with $p = 0.5$, $\delta = \frac{\pi}{2}$, $\beta_1 = 2\pi$, $\beta_2 = \frac{\pi}{6}$, $\beta_4 = \pi$ and $\varepsilon = \frac{1}{168}$

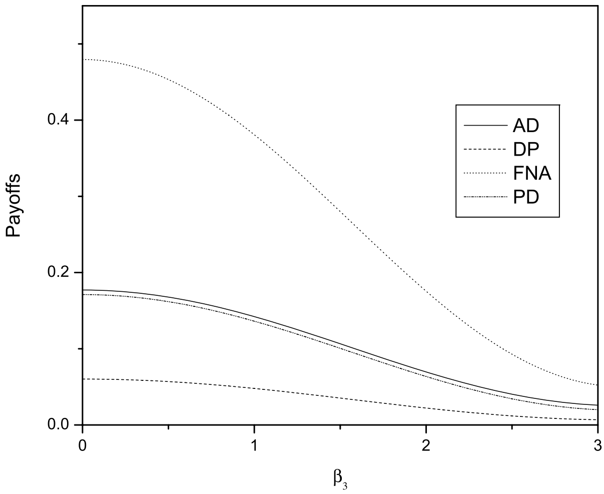
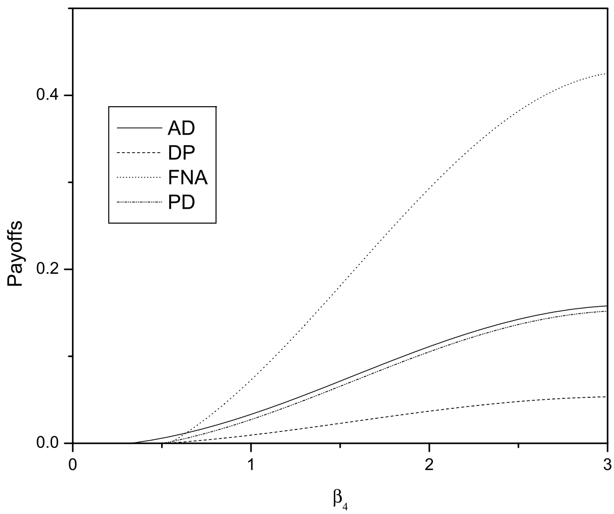


Fig. 6 The expected payoffs for a single game of sequence AAB are plotted as a function of the quantum phase angle, β_4 for amplitude damping channel, depolarizing channel, phase damping channel and FNA results with $p = 0.5$, $\delta = \frac{\pi}{2}$, $\beta_1 = \frac{\pi}{4}$, $\beta_2 = \frac{\pi}{4}$, $\beta_3 = \frac{\pi}{4}$ and $\varepsilon = \frac{1}{168}$



The results given in (17) are obtained at the maximum payoffs condition (i.e. $\beta_2 = \beta_3 = \pi - 2\delta$ and $\beta_1 = \beta_4 = -2\delta$) as given in Ref. [19]. From Fig. 7, one can see that for $\varepsilon = \frac{1}{168}$, the amplitude damping channel leads to the winning game. It is also seen that the quantum enhancement reduces when we play a repeated sequence of AAB on the maximally entangled initial state, because of the destructive interference of phase dependent terms. Whereas the depolarizing channel corresponds to the loosing game. However, from the inset figure, it is seen that in the case of amplitude damping channel for the values of decoherence parameter p in the range $0.5 < p < 1$, the repeated sequence AAB leads to a negative payoff. Furthermore, it is clear from (17) that the phase damping channel does not influence the game. The results for a single sequence of game B under the action of three channels are

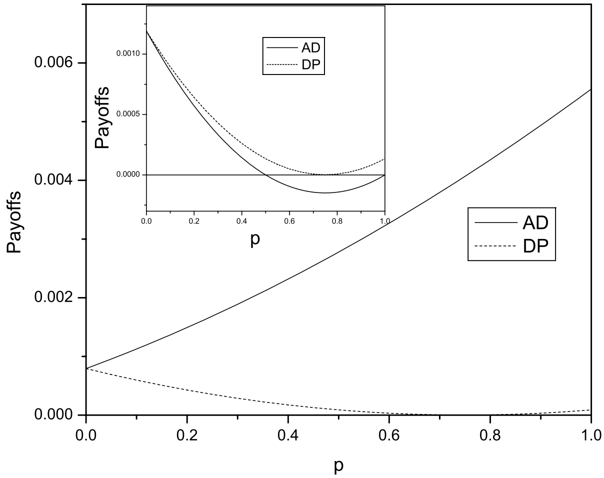


Fig. 7 The expected payoffs for a series of sequence AAB are plotted as a function of the decoherence parameter, p for amplitude damping and depolarizing channels with $\varepsilon = \frac{1}{168}$ (for *inset figure*, $\varepsilon = \frac{1}{112}$)

obtained as

$$\begin{aligned}
\langle \$^{\mathrm{AD}} \rangle &= \frac{1}{30}[2 + p\{-7 - 20\varepsilon + p(-29 + 22p)\}] \\
\langle \$^{\mathrm{DP}} \rangle &= \frac{1}{135}(3 - 4p)^2 \\
\langle \$^{\mathrm{PD}} \rangle &= \frac{1}{15}
\end{aligned} \tag{18}$$

In case of sequence BB, the payoffs become

$$\begin{aligned}
\langle \$^{\mathrm{AD}} \rangle &= \frac{1}{400}[13 - 2p\{67 + 2p(51 - 64p + 22p^2)\} \\
&\quad + 10\varepsilon\{2 + p(-11 - 62p + 44p^2)\}] \\
\langle \$^{\mathrm{DP}} \rangle &= \frac{1}{32400}[(3 - 4p)^2\{117 + 180\varepsilon + 88(3 - 2p)p\}] \\
\langle \$^{\mathrm{PD}} \rangle &= \frac{13}{400} + \frac{\varepsilon}{20}
\end{aligned} \tag{19}$$

Similarly, the payoffs for BBB sequence of the game can be written as

$$\begin{aligned}
\langle \$^{\mathrm{AD}} \rangle &= (0.017 - 0.41p - 0.13p^2 + 0.45p^3) \\
&\quad + (0.03 - 1.11p + 0.44p^2 - 2.66p^3)\varepsilon \\
\langle \$^{\mathrm{DP}} \rangle &= (0.017 + 0.01p - 0.13p^2 + 0.10p^3) \\
&\quad + (0.03 + 0.15p - 0.73p^2 + 0.83p^3)\varepsilon \\
\langle \$^{\mathrm{PD}} \rangle &= 0.017 + 0.03\varepsilon
\end{aligned} \tag{20}$$

One can easily check that by setting $p = 0$, in (17)–(20), the results of Ref. [19] can be reproduced. In Figs. 8 and 9, we have plotted the expected payoffs as a function of the

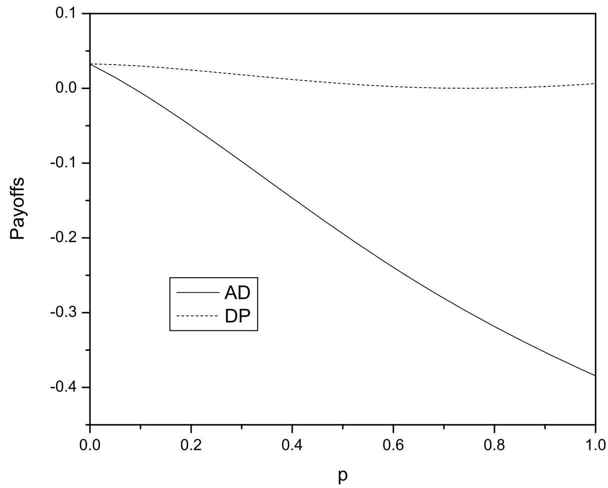


Fig. 8 The expected payoffs for the game sequence BB are plotted as a function of the decoherence parameter, p for amplitude damping and depolarizing channels with $\varepsilon = \frac{1}{112}$

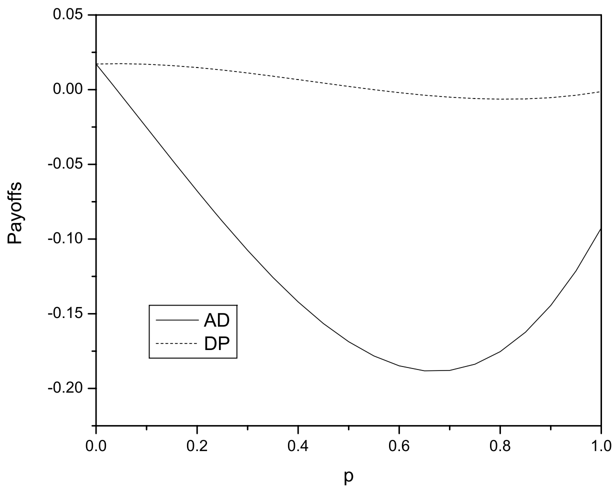


Fig. 9 The expected payoffs for the game sequence BBB are plotted as a function of the decoherence parameter, p for amplitude damping and depolarizing channels with $\varepsilon = \frac{1}{112}$

decoherence parameter, p for $\varepsilon = \frac{1}{168}$ for B's game sequences BB and BBB respectively. It can be inferred from the results for game B's sequences that the behavior of sequences B and BB is similar for amplitude damping and depolarizing channels. It is seen that the games are losing ones for both the channels. However, for higher values of decoherence parameter p, the payoff starts increasing in case of depolarizing channel. We have also calculated the results for the series of game A, the payoff under the influence of amplitude damping channel becomes $-\frac{3}{32}\varepsilon p$. As both ε and p are positive, the negative sign ensures that for any value of decoherence parameter p, the series of game A remains a losing one. However, the payoffs obtained for the series of game A under the action of depolarizing and phase damping channels are zero.

5 Conclusion

We study the history dependent quantum Parrondo's games under the effect of decoherence being played in different sequences for amplitude damping, depolarizing and phase damping channels. It is seen that the payoffs are enhanced in the presence decoherence for maximally entangled state in case of a single game of the sequence AAB for amplitude damping channel. The decoherence causes constructive interference of quantum phases that leads to the enhancement of payoff in comparison to the undecohered case. Whereas in case of the depolarizing and the phase damping channels, phases interfere destructively that results into a decrease in payoffs (a losing game sequence). It is also seen that the payoffs are significantly damped against the quantum phases in the presence of decoherence.

Furthermore, it is seen that for repeated games of sequence AAB, under the influence of amplitude damping channel the game becomes a winning game. However, the quantum enhancement in payoff reduces in this case as compared to the single game of sequence AAB. Whereas under the influence of depolarizing channel the game becomes a loosing game. It is shown that the games B and BB behave similarly for amplitude damping and depolarizing channels. It is seen that the repeated games of A are only influenced by the amplitude damping channel and it always remain a losing game. Furthermore, it is also seen that every sequence played repeatedly remains unaffected under the influence of phase damping channel.

Acknowledgement One of the authors (Salman Khan) is thankful to World Federation of Scientists for partial financial support under the National Scholarship Program for Pakistan.

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